

Problems for Seminar 7

Problem 1

- Do Problem 8.2 (except e) in the book by Church and Ware.

Problem 2

- Do Problem 8.4 in the book by Church and Ware.

Problem 3

- Consider a Bertrand duopoly with differentiated goods. Let p_1 denote Firm 1's price and p_2 denote Firm 2's price. The demand for Firm 1's good is given by $q_1 = 7 - 3p_1 + p_2$, and the demand for Firm 2's good is given by $q_2 = 7 - 3p_2 + p_1$. Both firms' marginal costs are constant and equal to one, and there are no fixed costs.
 - Calculate the Bertrand-Nash equilibrium.
 - How much does each firm produce in the Bertrand-Nash equilibrium?
 - What are the equilibrium profits?
 - Suppose the firms could collude perfectly and choose p_1 and p_2 so as to maximize the sum of the two firms' profits. What prices would they choose? *Hint:* Also when colluding, the firms produce both goods.

Problem 4 (inspired by Vives (Journal of Economic Theory, 1985))

This problem is deliberately a bit more challenging than most of the other problems that we solve in the course, and to carry out all the calculations may require quite a lot of work. However, even if you think the calculations are hard and tedious, it's important that you understand the principles behind the calculations. The last question (e) also concerns an interesting and important result about the relative efficiency of the Cournot model and the Bertrand model.

- Consider a market with two firms. The indirect demand functions for the firms' goods are

$$p_1 = \alpha - q_1 - \gamma q_2,$$

$$p_2 = \alpha - q_2 - \gamma q_1,$$

where α and γ are parameters satisfying $\alpha > 0$ and $-1 < \gamma < 1$. The firms have the same cost function, which is given by

$$C(q_i) = cq_i,$$

where c is a parameter satisfying $0 \leq c < a$.

- Interpret the parameter γ . For positive (resp. negative) values of γ , do the consumers consider the two goods to be complements or substitutes?
- Calculate the Cournot-Nash equilibrium. What is the market price for each good in this equilibrium? Are q_1 and q_2 strategic substitutes or strategic complements?
- Invert the two indirect demand functions so that you get two direct demand functions.
- Calculate the Bertrand-Nash equilibrium. What is the market price for each good in this equilibrium? Are p_1 and p_2 strategic substitutes or strategic complements?
- Which model (quantity setting or price setting) gives rise to the lowest market price?