

## Extra Problems for Seminar 14

### Problem 1

Consider the following version of Hotelling's linear city model with two firms that produce and sell a homogeneous good. The firms cannot choose the price of the good; instead this is fixed at £5. The firms can, however, choose where to locate. In particular, the firms can position themselves at one of four different locations. These locations lie on a line, as follows:

A    B    C    D.

All adjacent locations are equidistant from each other, and this distance is assumed to be one mile. At location A there are 1,000 costumers; at location B there are 2,000 costumers; at location C there are 3,000 costumers; and at location D there are 2,000 costumers. Each costumer buys either one unit of the good or nothing at all. A costumer's gross surplus from consuming the good equals £15. However, in order to obtain the good the costumer must pay the above mentioned price of £5. In addition, the costumer must incur a travel cost in order to get from where they are to the location where the nearest firm is and back. The travel cost is £2 per mile. This means that, for example, for a costumer who is at A: travelling to a firm that is also located at A doesn't cost anything; travelling to a firm at B and back costs £4; and so on. A costumer chooses to buy one unit if the gross surplus of £15 exceeds the sum of the price (£5) and the travel cost; otherwise he or she does not buy anything. If travelling to one firm is cheaper than travelling to the other, a costumer chooses to buy from the nearest one. It is assumed that if a costumer or a number of costumers are indifferent between two firms, the firms share these costumers equally between themselves. The firms can produce the good at a cost of £1 per unit.

- a) Assume that the two firms choose their locations simultaneously (and that the consumers thereafter, knowing the locations, make their optimal choices). Identify the (Nash) equilibrium of the game between the two firms. Explain why any other pair of locations than the ones you have identified cannot be an equilibrium. Also calculate the firms' profits and the consumer surplus for the different categories of costumers as well as aggregate consumer surplus.
- b) Give an example of one configuration of locations (which is not necessarily an equilibrium) that would yield a total consumer surplus that is larger than the one in a). Explain (in economic terms) why this consumer surplus is larger.

## Problem 2

Solve the four-firm version of the Hotelling's linear city model (with exogenous prices but endogenous locations) that is described in Lecture Notes 15.