Industrial Economics (EC3313) 2006/07

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Bertrand and Stackelberg lectures: Short version

Bertrand model

- Price competition, homogeneous goods
  - => p=MC
- Marginal cost pricing with only two firms:
  - => "Bertrand paradox"
- Key assumptions behind the p=MC result:
  1. No capacity constraints.
     - We won't model capacity constraints in this course, but you must understand why the assumption is crucial.
  2. There is a single period.
     - We'll study models with many periods later in the course — in such models the marginal-cost-pricing result does not need to hold.
  3. The firms produce identical goods.
     - If we assume that the goods are (to some extent) differentiated, then we can get a positive mark-up.
     - In particular, the Lerner index for Firm 1 in this model can be written
       $$L_1 \equiv \frac{p_1^B - c_1}{p_1^B} = \frac{1}{\varepsilon_{11}},$$
       where ε_{11} is the own-price elasticity of demand for Good 1:
       $$\varepsilon_{11} = -\frac{p_1}{q_1}\frac{\partial q_1}{\partial p_1}.$$
     - In the problem sets you'll solve many Bertrand models with differentiated goods.

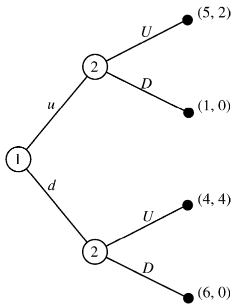
Strategic substitutes and complements

- We say that:
  - If the best-response functions are *downward-sloping*, then s_1 and s_2 are **strategic substitutes**.
  - If the best-response functions are *upward-sloping*, then s_1 and s_2 are **strategic complements**.
- Typically,
  - In a Cournot model, the quantities are strategic substitutes.
  - In a Bertrand model, the prices are strategic complements.
- Whether s_1 and s_2 are strategic substitutes or complements is crucial for the effect of comparative statics.
  - *Comparative statics*: What is the effect on the endogenous variables if there is a change in an exogenous parameter?

The Extensive Form Representation of a Game

Information provided by an extensive form game:

1. The number of players.
2. When each player can take an action.
3. What actions are available for a player when it's her turn to move.
4. Each player's payoff for all possible outcomes of the game.



Subgame perfect Nash equilibrium (SPNE)

- A **subgame** is defined as a single node and all the branches and nodes and payoffs that flow from that node.
  - The full game counts as a subgame.

- **Subgame perfect Nash equilibrium (SPNE)**: a strategy profile that is a NE in every subgame.

How to identify the SPNE:

- In a game with a finite horizon, we can find the subgame perfect Nash equilibria by **backward induction**:
  1. Identify the smallest possible subgames.
  2. Ask: for each of these subgames, what is the NE (or, if relevant, the optimal choice of the single player) in that subgame?
  3. Replace these subgames with the implied payoffs, making them terminal nodes of the new reduced-form game. Then go back to 1.

The Stackelberg model

- A famous example of strategic behavior:
  - The **Stackelberg model**.

- This is exactly like a Cournot duopoly, but:
  - Firm 1 chooses its output first.
  - Firm 2 observes Firm 1's output before it chooses its own output.

- Profits for the two firms:

$$\pi_1(q_1, q_2),$$

$$\pi_2(q_1, q_2).$$

- Recipe:
  1. Solve the follower's (firm 2's) problem:

$$\max_{q_2} \pi_2(q_1, q_2) \qquad \Rightarrow q_2^*(q_1)$$

  2. Plug $q_2^*(q_1)$ into the leader's (firm 1's) profit function:

$$\pi_1(q_1, q_2^*(q_1)).$$

  3. Maximize this w.r.t. q_1, to get the leader's equilibrium output:

$$\max_{q_1} \pi_1(q_1, q_2^*(q_1)) \Rightarrow q_1^*.$$

  4. To get the follower's equilibrium output, plug q_1^* into $q_2^*(q_1)$:

$$q_2^*(q_1^*) = q_2^*.$$

- See also figure on whiteboard.

- Example:
  - Inverse demand:
  $$p = 1 - q_1 - q_2.$$
  - Each firm has a constant marginal cost of c ($0 \le c < 1$), and there are no fixed costs.
  - Firm 1 is the leader and firm 2 is the follower.

- Solve the follower's (firm 2's) problem:
$$\max_{q_2} (1 - c - q_1 - q_2)\, q_2.$$
FOC:
$$-q_2 + 1 - c - q_1 - q_2 = 0 \qquad \Rightarrow q_2^*(q_1) = \frac{1 - c - q_1}{2}.$$

- Plug $q_2^*(q_1)$ into the leader's (firm 1's) profit function:
$$\begin{aligned} \pi_1(q_1, q_2^*(q_1)) &= [1 - c - q_1 - q_2^*(q_1)]\, q_1 \\ &= \left[1 - c - q_1 - \frac{1 - c - q_1}{2}\right] q_1 \\ &= \left[\frac{1 - c - q_1}{2}\right] q_1 \end{aligned}$$

- Maximize this w.r.t. q_1, to get the leader's equilibrium output:
$$\max_{q_1} \left[\frac{1 - c - q_1}{2}\right] q_1 \qquad \Rightarrow q_1^* = \frac{1 - c}{2}.$$

- To get the follower's equilibrium output, plug q_1^* into $q_2^*(q_1)$:
$$q_2^*(q_1^*) = q_2^* = \frac{1 - c - q_1^*}{2} = \frac{1 - c}{4}.$$

- More general results (which can be proven):
  - With linear demand and cost, the leader produces the monopoly output and the follower half of that.
  - Larger aggregate output than in Cournot duopoly (which is good for total surplus and bad for industry profits)
  - The leader is better off than the follower, thanks to its ability to make a strategic move (this result due to strategic substitutes — would be reversed with strategic complements).