



Figure 1: Joseph Bertrand (1822-1903)

**Lecture 6:** Bertrand comp. (C&W, parts of Chs 8 and 15)

## Today's Agenda

- Finish Lecture 5:
  - Cournot competition and the Herfindahl-Hirschman index.
- Lecture 6: Bertrand competition.
  - Bertrand duopoly, **Homogeneous** good.
  - Bertrand duopoly, **heterogeneous** (differentiated) goods.
  - Strategic substitutes and complements.

## A Duopoly Version of the Bertrand Model

- Two firms produce identical products and have the same constant marginal cost ( $c$ ).
- No other producers are able to enter the market.
- The firms interact just once and they make their pricing decisions simultaneously.
- The firms choose prices (and the consumers choose from which firm to buy).

### The normal (or strategic) form of this game

- Set of players:  $N = \{1, 2\}$ .
- Strategy sets:  $S_1 = S_2 = [0, \infty)$ .
  - Denote an element of  $S_i$  (i.e., firm  $i$ 's price) by  $p_i$ .
- Payoff functions (the firms' profits):

$$\pi_1(p_1, p_2) = (p_1 - c) q_1(p_1, p_2),$$

$$\pi_2(p_1, p_2) = (p_2 - c) q_2(p_1, p_2),$$

where

$$q_1(p_1, p_2) = \begin{cases} a & \text{if } p_1 < p_2 \\ \frac{a}{2} & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2, \end{cases}$$

and

$$q_2(p_1, p_2) = a - q_1(p_1, p_2).$$

### A Nash equilibrium of this Bertrand model

- The pair of prices  $(p_1^B, p_2^B)$  is a **Bertrand-Nash equilibrium** if neither firm can increase its profits by unilaterally choosing some other price, *given the equilibrium price of its rival*:

$$\pi_1(p_1^B, p_2^B) \geq \pi_1(p_1, p_2^B) \quad \text{for every } p_1,$$

$$\pi_2(p_1^B, p_2^B) \geq \pi_2(p_1^B, p_2) \quad \text{for every } p_2.$$

- This is often called a Bertrand (or Bertrand-Nash) equilibrium.
  - But think of it as a Nash equilibrium of the Bertrand model.
- As before when we studied the Cournot model:
  - We stick to a *single equilibrium concept* but vary the rules of the game.

- **Claim:** The only Nash equilibrium is  $(p_1^B, p_2^B) = (c, c)$ .

- **Proof.** *Key idea:* if a firm chose a price above  $c$ , then its rival could choose a price that is slightly lower but still above  $c$ , thereby grabbing the whole market and make a profit. Therefore, no price can be above  $c$  in an equilibrium.

- *More formally:* Go through all different possibilities and show that, for all strategy profiles except  $(p_1^B, p_2^B) = (c, c)$ , at least one firm wants to deviate.

- A strategy profile where one or both firms charge a price **below**  $c$  can clearly not be part of an equilibrium.

- \* For then at least one firm would earn negative profits, which it can avoid by deviating to a higher price.

- If  $(p, p_2) = (c, c)$ , then the firms share the market but sell at their unit cost, so each firm earns zero profits.

- \* If one of the firms deviated to a **lower** price, it would grab the whole market. But it would have to sell at a price that is below its unit cost, so it would make a loss. Therefore, a firm does not have an incentive to deviate downwards.

- \* If one of the firms deviated to a **higher** price, then it would not be able to sell anything and would therefore earn zero profits. Therefore, a firm does not have an incentive to deviate upwards (it wouldn't get strictly higher profits).

- \* We can conclude that  $(p_1, p_2) = (c, c)$  is indeed a Nash equilibrium.

- There are three cases that are left to consider (the cases where 1 and 2 have reversed roles are qualitatively identical to the ones below, so they can safely be ignored):

a)  $c = p_1 < p_2$ ;

b)  $c < p_1 < p_2$ ;

c)  $c < p_1 = p_2$ .

- In case a), Firm 1 can deviate profitably by charging some price  $p'_1$  in the interval

$$c < p'_1 < p_2$$

(it doesn't lose any customers but sell at a higher price).

- In case b), Firm 1 can deviate profitably by charging some price  $p'_1$  in the interval

$$p_1 < p'_1 < p_2$$

(it doesn't lose any customers but sell at a higher price). Firm 2 can also deviate profitably by charging some price  $p'_2$  in the interval

$$c < p'_2 < p_1$$

(it grabs the whole market and sells at a price above its unit cost).

- In case c), either firm can deviate profitably by undercutting its rival slightly (charging some price  $p_1 - \varepsilon$ , where  $\varepsilon$  is small but positive). By doing so, it can sell to twice as many customers at a price that is only slightly lower than before.

- **Conclusion:** when the firms are assumed to choose prices (instead of quantities), the only NE is such that both firms set price equal to MC, just as under perfect competition.

- The **Bertrand Paradox:**

- According to the model, two firms are enough to eliminate market power.

- Our intuition and observation of real world markets would suggest otherwise.

## Variants of the model that don't give rise to the paradox

- **Capacity constraints:**

- Homogeneous products but limitations to how much a firm can produce.
- \* We will not solve this model in the course.

- **Product differentiation:**

- The products are not identical, so a firm can sell a positive quantity even if it charges a higher price than its rival.
- Let's study such a model!

## Bertrand with differentiated goods

- Two firms produce goods that are **imperfect substitutes**.

- Imperfect substitutes  $\Rightarrow$  Consumers buy some positive quantity of Good 2 also when  $p_1 < p_2$ , because Good 1 is not a perfect substitute for Good 2.

- Demand for Good 1 is given by  $q_1(p_1, p_2)$ , where this function varies smoothly with changes in  $p_1$  and  $p_2$  (in contrast to the jumping behavior we assumed above).

- Similarly, demand for Good 2 is given by  $q_2(p_1, p_2)$ .

- An increase in own price lowers demand, and an increase in the rival's price increases demand.

$$\frac{\partial q_1}{\partial p_1} < 0 \quad \text{and} \quad \frac{\partial q_1}{\partial p_2} > 0$$

- Let's use **best-response functions**,  $B_1(p_2)$  and  $B_2(p_1)$ , to characterize the Bertrand-Nash equilibrium.

- Recall: Firm 1's best-response function tells us what price is optimal for Firm 1 if Firm 2 chooses some price  $p_2$ .
- So Firm 1's best-response is the solution to the following problem (where  $p_2$  is treated as fixed and given):

$$\max_{p_1 \geq 0} (p_1 - c_1) q_1(p_1, p_2).$$

- FOC:

$$q_1 + (p_1^B - c_1) \frac{\partial q_1}{\partial p_1} = 0.$$

- Quite generally in the Bertrand model:  $B_1(p_2)$  is upward-sloping —  $p_1$  and  $p_2$  are **strategic complements**.

\* See figure on whiteboard.

## The Lerner index under Bertrand

- Rewriting Firm 1's FOC from previous slide yields

$$\frac{p_1^B - c_1}{p_1^B} = - \frac{1}{\frac{p_1^B}{q_1} \frac{\partial q_1}{\partial p_1}}.$$

The own-price elasticity of demand for Good 1:

$$\varepsilon_{11} = - \frac{p_1}{q_1} \frac{\partial q_1}{\partial p_1}.$$

Using this, we get

$$\frac{p_1^B - c_1}{p_1^B} = \frac{1}{\varepsilon_{11}}.$$

## Observations:

1. Each firm exercises market power:  $p^B > c_1$ .
2. Market power limited by the demand elasticity:  $\varepsilon_{11} \nearrow \Rightarrow \text{mark up} \searrow$ . [The magnitude of  $\varepsilon_{11}$  depends on how close substitutes the two goods are.]

## Strategic substitutes and complements

- Consider a game with two players, 1 and 2.
- The players' strategies are denoted  $s_1$  and  $s_2$ .
  - For example,  $s_1$  and  $s_2$  could be quantities (Cournot) or prices (Bertrand).
- The players' best-response functions are denoted  $B_1(s_2)$  and  $B_2(s_1)$ .
- We say that:
  - If the best-response functions are *downward-sloping*, then  $s_1$  and  $s_2$  are **strategic substitutes**.
  - If the best-response functions are *upward-sloping*, then  $s_1$  and  $s_2$  are **strategic complements**.

- Typically,
  - In a Cournot model, the quantities are strategic substitutes.
  - In a Bertrand model, the prices are strategic complements.
- Whether  $s_1$  and  $s_2$  are strategic substitutes or complements is crucial for the effect of comparative statics.
  - *Comparative statics*: What is the effect on the endogenous variables if there is a change in an exogenous parameter?

- Suppose Firm 1's marginal cost decreases (exogenously).
  - **Cournot competition:**
    - \*  $B_1(s_2)$  and  $B_2(s_1)$  downward-sloping.
    - \*  $B_1(s_2)$  shifts outwards [lower MC makes optimal output larger for any given behavior of rival]
    - \*  $q_1^* \nearrow$  and  $q_2^* \searrow$  [see figure on whiteboard!]
  - **Bertrand competition:**
    - \*  $B_1(s_2)$  and  $B_2(s_1)$  upward-sloping.
    - \*  $B_1(s_2)$  shifts leftwards [lower MC makes optimal price lower for any given behavior of rival]
    - \*  $p_1^* \searrow$  and  $p_2^* \searrow$  [see figure on whiteboard!]

## Conclusion

- The distinction between strategic substitutes and complements is crucial for understanding many economic theories.
- Differences between the Cournot model and the Bertrand model are typically not due to quantity setting respectively price setting per se, but rather the fact that the strategic variables are strategic substitutes in one of the models and strategic complements in the other.