

**Lectures 13 & 14:** Empirical tests of oligopoly  
(C&W, parts of Chapters 8 & 12)

## Today's Agenda

- The Conduct Parameter
- Two ways of thinking about the conduct parameter
  - Conjectural Variations
  - The Competition Parameter
- Identifying the conduct parameter
  - The Bresnahan-Lau approach
- Some empirical results

## The Conduct Parameter

- So far in the course:
  - A number of theories about oligopolistic behavior.
- Different models imply different degrees of market power and competition.
  - Bertrand with homogeneous goods [ $\Rightarrow$  marginal cost pricing]
  - Cournot with homogeneous or heterogeneous goods [ $\Rightarrow$  some market power — how much depending on the number of firms and on the degree of product differentiation — but full collusion not possible]
  - Infinitely repeated Cournot (or Bertrand) game [ $\Rightarrow$  full collusion if firms are sufficiently patient.]

- Question: Which model is right, given some market environment?

- As a first step towards answering this, we can summarize all the models that we have studied in a single equation, the so-called **supply relationship**:

$$D(Q) + \lambda D'(Q) q_i - C'(q_i) = 0, \quad (1)$$

where

- $D(Q)$  ( $= p$ ) is the indirect demand function,
  - $C(q_i)$  is a firm's cost function,
  - $Q = \sum_{i=1}^n q_j$  is total output in the industry,
  - $q_i$  is Firm  $i$ 's output
  - and  $\lambda$  is the so-called **conduct parameter**.
- Different values of the conduct parameter yield the outcome of different oligopoly models, and our objective will be to estimate the value of the conduct parameter empirically.

- In particular we have:
  - $\lambda = 0$  corresponds to price taking behavior, as this yields  $p=MC$ .
  - $\lambda = 1$  corresponds to Cournot behavior, as this yields the FOC for a Cournot oligopolist.
  - $\lambda = n$  corresponds to collusive behavior, as this yields a condition that defines the optimal cartel output.
    - \* To see this, note that under symmetry we have  $nq_i = Q$ , and the FOC
$$D(Q) + D'(Q) Q - C'(q_i) = 0$$
defines Firm  $i$ 's optimal cartel output.
- More generally, larger values of  $\lambda$  yield a higher price-MC margin, and  $\lambda$  can therefore be thought of as a measure of market power.

- We can also understand the role of  $\lambda$  by relating it to the Lerner index, which we recall is defined as

$$LI = \frac{p - MC}{p}.$$

- Rewriting the supply relationship (1) (imposing symmetry) yields

$$LI = -\frac{\lambda D'(Q) q}{D(Q)} = -\frac{\lambda Q D'(Q)}{n D(Q)} = \frac{\lambda}{n \varepsilon},$$

where

$$\varepsilon \equiv -\frac{dQ}{dp} \frac{p}{Q}.$$

- Interpretation:
  - The LI is a measure of market power that runs from 0 to 1.
  - Its value in this model is increasing in the conduct parameter  $\lambda$  (which is consistent with the results above).

## Two ways of thinking about the conduct parameter

- Before investigating how we can estimate the conduct parameter, we will here study two different (but, as it turns out, equivalent) interpretations of this parameter.
- In particular the first interpretation, the conjectural variations approach, is an often used tool in industrial economics and is therefore worthwhile knowing about in its own right, not only as a way of thinking about the conduct parameter.

### Conjectural Variations

- Not proper game theory, but useful for empirical studies.

- Firms produce and sell a homogeneous good, they choose quantities, the indirect demand function is given by  $D(Q)$ , and a firm's cost function is given by  $C(q_i)$ .

- **Key assumption:**

- Each Firm  $i$  believes (forms a conjecture) that if it changes its output, the rival, Firm  $j$ , will *react* to some extent by changing its own output:

$$\frac{dq_j}{dq_i} = v_{ij} \quad \text{for all } j \neq i.$$

- The parameters  $v_{ij}$  are called **conjectural variations parameters**.
- Note: The choices are made simultaneously and the firms interact only once, so expecting a response from your rival doesn't make sense (this is why it's not proper game theory). Ignore this (think of it as a reduced form of a richer model).

- Consider Firm  $i$ 's maximization problem:

$$\max_{q_i \geq 0} D(Q) q_i - C(q_i).$$

- Now take the FOC — exactly as before except that we now must take the reactions (the  $v_{ij}$ s) into account:

$$D(Q) + D'(Q) \left[ 1 + \sum_{j \neq i} \frac{dq_j}{dq_i} \right] q_i - C'(q_i) = 0$$

or

$$D(Q) + D'(Q) \left[ 1 + \sum_{j \neq i} v_{ij} \right] q_i - C'(q_i) = 0$$

or, assuming symmetry,

$$D(Q) + [1 + (n - 1)v] D'(Q) q_i - C'(q_i) = 0.$$

- We see that

$$1 + (n - 1)v = \lambda, \quad (2)$$

where  $\lambda$  is the conduct parameter defined above.

- Note that:

- $v = -1/(n - 1)$  corresponds to price taking behavior, as this yields  $p=MC$ .
- $v = 0$  corresponds to Cournot behavior, as this yields the FOC for a Cournot oligopolist.
- $v = 1$  corresponds to collusive behavior, as this yields a condition that defines the optimal cartel output.

- Conclusion: the conjectural variation story, with firms being able to react to each other's choices, is one way of thinking about the conduct parameter.

- The exact relationship between the conduct parameter and the conjectural variation parameter is given by (2).

## The “Competition Parameter”

- This is another way of thinking about the conduct parameter.

- It is conceptually different from the conjectural variation approach.
- However, it turns out that with symmetric firms this approach and the conjectural variation approach are formally equivalent (in a problem set you will see that this is not true in an asymmetric model).

- **Key assumption:**

- Each firm is assumed to maximize a weighted sum of its own profits and the rivals' joint profits, where the weight for the latter is the parameter  $\mu$ :

$$D(Q)q_i - C(q_i) + \mu \left[ \sum_{j \neq i} D(Q)q_j - C(q_j) \right]$$

- The weight  $\mu$  is a fixed parameter, which we will call the “**competition parameter**” (in the literature it has been called the “coefficient of cooperation” and the “coefficient of effective sympathy”).

- Even though I call it a “weight”, it's not necessarily the case that  $0 \leq \mu \leq 1$ .

- One can show that:

- $\mu = -1/(n - 1)$  corresponds to price taking behavior, as this yields  $p=MC$ .
- $\mu = 0$  corresponds to Cournot behavior, as this yields the FOC for a Cournot oligopolist.
- $\mu = 1$  corresponds to collusive behavior, as this yields a condition that defines the optimal cartel output.

- That is, at least for these three cases,  $\mu = v$ , and the role of the “competition parameter”  $\mu$  is exactly the same as that of the conjectural variation parameter  $v$ .

- Conclusion: the “competition parameter” story, with firms maximizing some weighted sum of their own profits and the rivals' profits, is another way of thinking about the conduct parameter.

- Formally, with symmetric firms this approach is equivalent to the conjectural variations approach (even though this is not obvious a priori).
- Therefore, the exact relationship between the “competition parameter” and the conjectural variation parameter is again given by (2).

## Identifying the conduct parameter

### The Problem

- We want to learn about the degree of competition in some market, and our measure of competition is the conduct parameter,  $\lambda$ .
- Only a limited amount of data are available:
  - The market price,  $p$ .
  - Total industry output,  $Q$ .
  - Consumer income or some other variable that shifts the demand curve,  $Z$ .
  - Price of a production factor (like the wage level or the price of energy or material),  $W$ .
  - The number of firms in the market,  $n$ .

- Each variable is observed in many consecutive periods (maybe yearly data).
- We have *no* data on the firms' costs or on firm specific outputs.
- The question: Can we identify and estimate the conduct parameter  $\lambda$ ?

### The Bresnahan-Lau approach

- The methodology explained below was introduced by Bresnahan (1982) and Lau (1982).
  - Here we will assume a linear demand function and quadratic cost function. However, one can use a similar methodology with, for a example, a demand function that is not linear.

- Thus, assume a linear inverse demand function and a quadratic cost function:

$$D(Q) = \delta_0 + \delta_1 Q + \delta_2 Z + \delta_3 QZ,$$

$$C(q_i) = \tau_{0i} + \tau_{1i} q_i + (\tau_2/2) q_i^2 + \tau_3 q_i W + \tau_4 W + \tau_5 W^2.$$

- Given these assumptions, we have

$$D'(Q) q_i = (\delta_1 + \delta_3 Z) q_i, \quad (3)$$

$$C'(q_i) \equiv MC_i = \tau_{1i} + \tau_2 q_i + \tau_3 W. \quad (4)$$

- Recall the supply relationship (allowing for firm specific cost functions and therefore asymmetries across firms in terms of output):

$$D(Q) + \lambda D'(Q) q_i - C'_i(q_i) = 0 \Leftrightarrow$$

$$p = MC_i - \lambda_i D'(Q) q_i. \quad (5)$$

- Plugging (3) and (4) into (5):

$$p = \tau_{1i} + \tau_2 q_i + \tau_3 W - \lambda (\delta_1 + \delta_3 Z) q_i.$$

This holds for all firms  $i$  — adding them up yields

$$np = \sum_{i=1}^n \tau_{1i} + \tau_2 Q + n\tau_3 W - \lambda (\delta_1 + \delta_3 Z) Q$$

or

$$\begin{aligned} p &= \frac{1}{n} \sum_{i=1}^n \tau_{1i} + \frac{\tau_2}{n} Q + \tau_3 W - \frac{\lambda \delta_1}{n} Q - \frac{\lambda \delta_3}{n} Z Q \\ &= \frac{1}{n} \sum_{i=1}^n \tau_{1i} + \frac{\tau_2 - \lambda \delta_1}{n} Q + \tau_3 W - \frac{\lambda \delta_3}{n} Z Q. \end{aligned}$$

- The relationship thus has the following form:

$$p = \alpha_0 + \alpha_1 Q + \alpha_2 Z + \alpha_3 W, \quad (6)$$

where

$$\alpha_0 = \tau_1 = \frac{1}{n} \sum_{i=1}^n \tau_{1i}, \quad (7)$$

$$\alpha_1 = \frac{\tau_2 - \lambda \delta_1}{n}, \quad (8)$$

$$\alpha_2 = -\frac{\lambda \delta_3}{n}, \quad (9)$$

$$\alpha_3 = \tau_3. \quad (10)$$

- We can estimate the supply relationship in (6) jointly with the demand function

$$p = \delta_0 + \delta_1 Q + \delta_2 Z + \delta_3 QZ$$

(adding an error term to each equation).

- Practical problem: The RHS variable  $Q$  is endogenous.
- Solution: Use  $Q$  from a previous period as an instrument.
- Estimating the two equations gives us estimates of the  $\delta$ s and the  $\alpha$ s.
- That is, we can identify all the demand parameters, i.e., all the  $\delta$ s.
  - The other parameters — the conduct parameter and the parameters in the cost function — are not, however, automatically identified.

- By studying the supply relationship (6) we see that we can recover  $\lambda$  if  $\delta_3 \neq 0$ . For then it follows from (9) that we can write  $\lambda$  as

$$\lambda = -\frac{\alpha_2 n}{\delta_3},$$

and we know  $n$  by assumption and we have estimates of  $\delta_3$  and  $\alpha_2$ .

- Once we know  $\lambda$ , we can infer  $\tau_2$  from (8):

$$\alpha_1 = \frac{\tau_2 - \lambda \delta_1}{n} = \frac{\tau_2 - \left(-\frac{\alpha_2 n}{\delta_3}\right) \delta_1}{n} \Rightarrow \tau_2 = \left(\alpha_1 - \frac{\alpha_2}{\delta_3} \delta_1\right) n,$$

where we know  $n$  and we have estimates of  $\delta_1$ ,  $\delta_3$ ,  $\alpha_1$  and  $\alpha_2$ .

- Finally, (10) gives us an estimate of  $\tau_3$ , and (7) gives us an estimate of  $\tau_1 \equiv \frac{1}{n} \sum_{i=1}^n \tau_{1i}$  (but not the individual  $\tau_{1i}$ s).

- Nor can we identify or estimate the cost function parameters that don't show up in the MC functions,  $\tau_{0i}$ ,  $\tau_4$  and  $\tau_5$ .

- Conclusion:

- We can recover the conduct parameter  $\lambda$  (as well as many other parameters) if  $\delta_3 \neq 0$ .
- If  $\delta_3 = 0$ , then (8) is the only equation that involves  $\lambda$ . However, this equation also contains  $\tau_2$ , which doesn't show up in any other equation. Therefore, we cannot separate  $\lambda$  from  $\tau_2$  — these two parameters are not identified.
- The economic interpretation of  $\delta_3 \neq 0$ :
  - The slope of demand function is a function of a shift variable (and we have data on this variable).

- Graphical analysis:

- Figure 1: shift only in the demand *intercept*.

- \* Low intercept: Perfect competition and monopoly both give rise to equilibrium  $E_1$ .
- \* High intercept: Perfect competition and monopoly both give rise to equilibrium  $E_2$ .
- \* Conclusion: If we observe only  $p$ ,  $Q$  and demand, we *cannot* infer whether we have perfect competition or monopoly.

- Figure 2: shift in the *slope* of demand.

- \* Perfect competition gives rise to the equilibrium  $E_1$  for both demand functions.
- \* Monopoly gives rise to equilibrium  $E_1$  with a steep demand but  $E_3$  with a flat demand.
- \* Conclusion: Even if we observe only  $p$ ,  $Q$  and demand, we *can* infer whether we have perfect competition or monopoly, as the two different market structures give rise to different behaviors (in terms of things we can observe).

## Some Empirical Results

### Brander and Zhang (1990) [C&W pp. 273-274]

- Assuming a duopoly we can rewrite the Lerner index with conjectural variations as follows:

$$\frac{p - MC_1}{p} = \frac{\lambda}{n\varepsilon} = \frac{s_1(v_1 + 1)}{\varepsilon},$$

where  $s_1$  is Firm 1's market share and  $\varepsilon$  is the demand elasticity.

- Solve for  $v_1$ :

$$v_1 = \frac{(p - MC_1)\varepsilon}{s_1 p} - 1. \quad (11)$$

- Of course, we can derive a similar expression for  $v_2$ .
- Recall the relationship between the conjectural variation parameter  $v_i$  and the conduct parameter  $\lambda_i$ :

$$\lambda_i = 1 + v_i.$$

- If we have estimates of price, marginal cost, demand elasticity, and market share, we can plug these into the formula (11).

- Brander and Zhang (1990) did this for American Airlines and United Airlines (33 duopoly routes out of Chicago):

	American	United
Point estimate of $v_i$	0.06	0.12
95% conf. interval	(-0.17, 0.30)	(-0.14, 0.38)

- Strong support for the Cournot behavior.
- See also Table 12.4 in C&W (page 447).

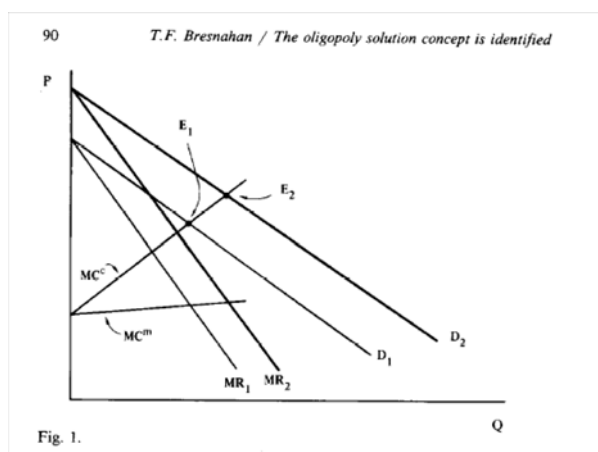


Figure 1:

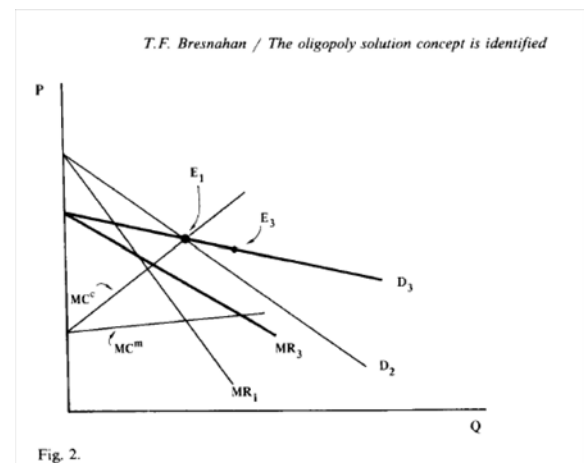


Figure 2: